Self-Phase-Modulation of Optical Pulses

From Filaments to Solitons to Frequency Combs

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SPM Generated Broadband Coherent Light

- A chance experimental discovery in the mid-sixties of far-reaching consequence.
- It took a several years to separate SPM from the myriad of other nonlinear effects associated with stimulated scattering.
- Broadband coherent light has enabled
  - Ultrafast science
  - Optical clock technology
- SPM of central significance in high-speed, long-distance fiber optical communication – whether it is a boon or a bane is still of debate.

Broadband Coherent Light
How Do We Make It?

- Modulators are limited to 10s of GHz.
- Laser modelocking can provide coherent broadband light in the active bandwidth of tunable lasers.
- 40 years ago, while studying stimulated light scattering (Raman, Brillouin) we discovered nonlinear spectral broadening of light that was seemingly unrelated to material excitation modes.
- This nonlinear broadening can markedly increase the spectral extent of coherent optical sources.
- The process proved to be an example of an optical self-action effect.

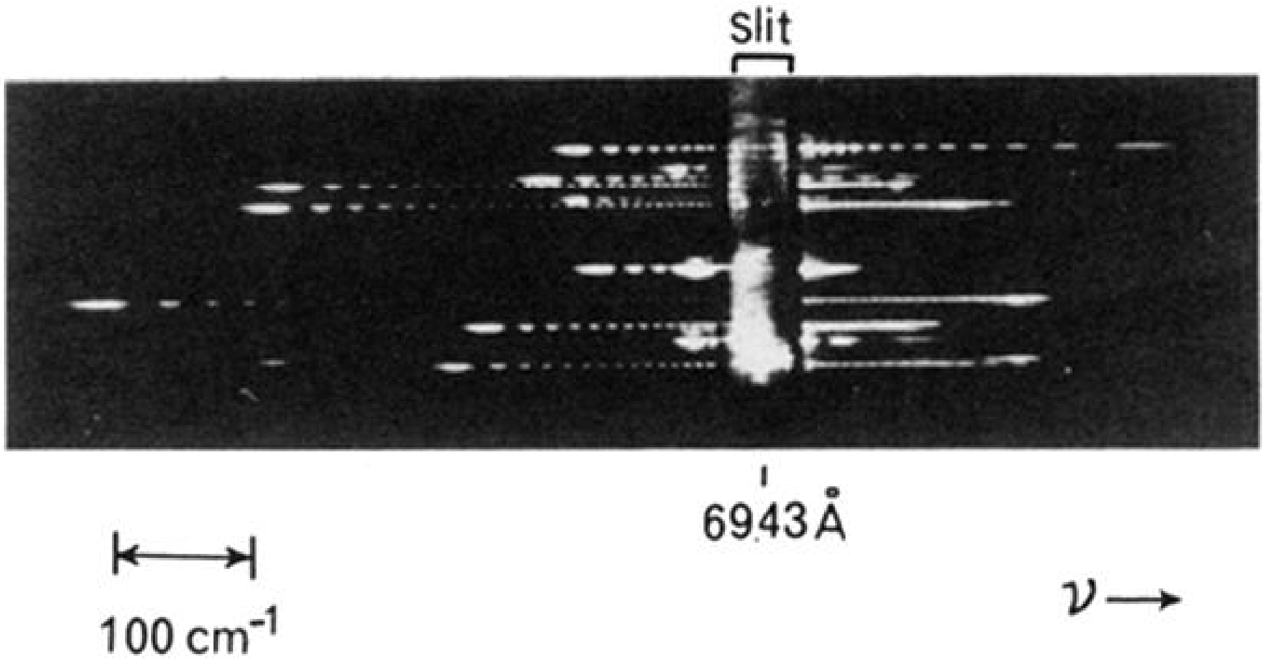
Optical Self-Action Effects

	Spatial	Temporal
Instabilities	Light-by-Light Scattering	Modulation Instability
Envelope Effects	Spatial Self-Phase Modulation Self-Focusing Whole beam Beam breakup Self-Trapping – Spatial Solitons	**Temporal Self-Phase Modulation – Self-Chirping** Self-Compression Self-Decompression – Self-Dispersion Temporal Solitons Self-Steepening
Combined	Light Bullets	

These are $\chi^{(3)}$ four wave mixing processes and are usually, but not always, elastic.

Early Experimental Observation of SPM

- Spectral broadening was first seen in small scale trapped filaments of light.
- The high intensity of reasonably long distance provided by self-focusing and self-trapping allowed the development of self-phase modulation.



Spectra of Small-scale filaments in CS_2

F. Shimizu, Phys. Rev. Lett. **14** , 1097 (1967).

Beats in the spectrum of each filament demonstrate the coherent nature of the process.

Other Early Experimental Observations of Spectral Enhancement

- B. P. Stoicheff, Phys. Lett. **7** 186 (1963).
- W. J. Jones and B. P. Stoicheff, Phys. Rev. Lett. **13**, 657 (1964).
- D. I. Mash, V. V. Morozov, V. S. Starunov, and I. L. Fabelinskii, ZETF Pisma **2**, 11 (1965); translation JETP Lett. , 25 (1965).
- N. Bloembergen and P. Lallemand, Phys. Rev. Lett. **16**, 81 (1966)
- R. G. Brewer, Phys. Rev. Lett. **19**, 8 (1967).
- H. P. Grieneisen, J. R. Lifsitz, and C. A. Sacchi. Bull. Am. Phys. Soc. **12**, 686 (1967).
- C. W. Cho. N. D. Foltz, D. H. Rank, and T. A. Wiggins, Phys. Rev. Lett. **18**, 107 (1967).
- A. C. Cheung, D. M. Rank, R. Y. Chiao, and C. H. Townes, Phys. Rev. Lett. **20** 786 (1968).
- C. A. Sacchi, C. H. Townes, and J. R. Lifsitz,, Phys. Rev. **174**, 438(1968).
- M. M. Denariez-Roberge and J.-P. E. Taran, Appl. Phys. Lett. **14**, 205 (1969). [Observed 2500 cm^{-1} spectral broadening.]
- R. R. Alfano and S. L. Shapiro, Phys. Rev. Lett. **24**, 584 (1970). [Observed 10,000 cm^{-1} spectral broadening.]

Physical Mechanisms for the Nonlinear Index of Refraction – The Optical Kerr Effect

$$n = n_0 + \delta n, \quad \delta n = 2n_2 < E^2 > \quad = n_2 \left| A \right|^2$$

- Pure electronic nonlinearity *à la* ABD&P
  - Homogeneous materials
  - Resonance nonlinearities
  - Quantum structures
  - Optical rectification, cascade nonlinear processes

- Motion of atoms and molecules – slow nonlinearities
  - Molecular alignment: anisotropic polarizability
  - Electrostriction
  - Thermal blooming
  - Photorefraction

Self-Phase Modulation

The equation for the slowly varying amplitude (A) without amplitude distortion or dispersion.

$$\left(\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t}\right) = ik\,\frac{n_2}{n_0}\left|A\right|^2 A$$

Solution
$$A(\tau, z) = A(\tau, 0)\,\mathrm{e}^{i\Phi_{NL}(\tau, z)}$$

$$\Phi_{NL}(\tau, z) = kz\,\frac{n_2}{n_0}\left|A(\tau, 0)\right|^2$$

$$\tau = t - z / v_g$$

- F. DeMartini, C. H. Townes, T. K. Gustafson, and P. L. Kelley, Phys. Rev. **164**, 312 (1967) [Includes self-steepening].
- F. Shimizu, Phys. Rev. Lett. **14** , 1097 (1967).

Self-Phase Modulation

Nonlinear frequency shift

$$\Omega(\tau, z) = -\frac{\partial \Phi_{NL}}{\partial \tau} = -kz\frac{n_2}{n_0}\frac{\partial |A(\tau)|^2}{\partial \tau}$$

Spectral Extent

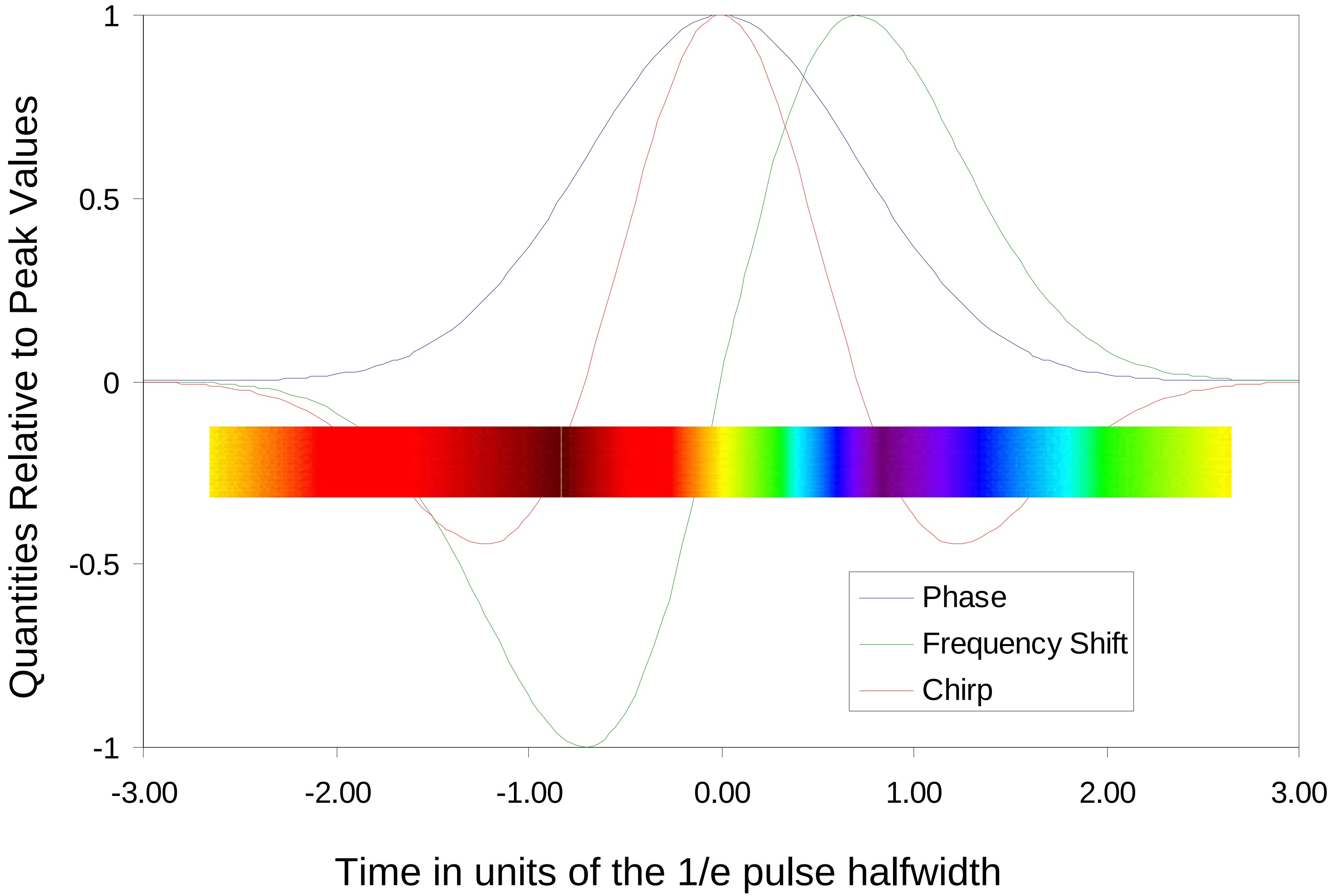
$$\Delta\Omega(z) = 2kz\frac{n_2}{n_0}\frac{\partial |A(\tau,0)|^2}{\partial \tau}\bigg|_{\max} \approx 2\frac{kz}{\tau_p}\frac{\delta n_{\max}}{n_0}$$

Chirp

$$C(\tau, z) = \frac{\partial \Omega(\tau, z)}{\partial \tau} = -kz\frac{n_2}{n_0}\frac{\partial^2 |A(\tau,0)|^2}{\partial \tau^2}$$

- The chirp has dimensions of Hz/s (perhaps best expressed in THz/ps).
- In this model, the pulse shape does not change in time, only the frequency spectrum. Fourier domain evolution.
- Frequency spectrum extent increases with increasing field amplitude and distance and with decreasing pulse length.

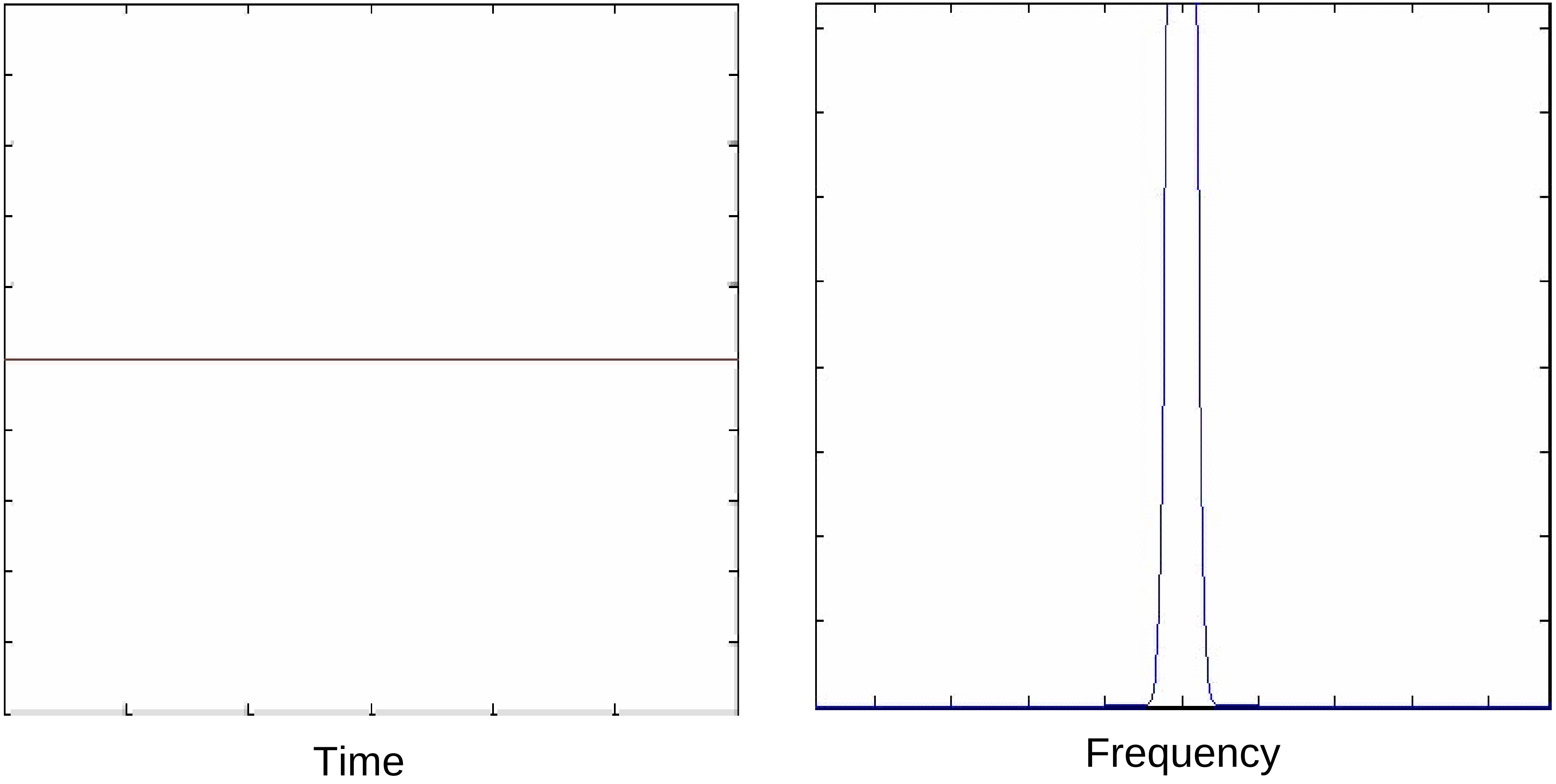
The Phase-Only Picture of Nonlinear Pulse Propagation



Frequencies can occur twice in the pulse. These two components can interfere constructively or destructively, leading to an amplitude modulated spectrum.

SPM Evolution of Phase, Instantaneous Frequency Change, Chirp and Spectrum with Distance

Pulse shape is Gaussian.



$$\Phi_{NL,\max} = 7\pi$$

The Nonlinear Schrödinger Equation: SPM and Dispersion

The Simplified NLSE

$$i\left(\frac{\partial A}{\partial z}+\frac{1}{v_g}\frac{\partial A}{\partial t}\right)-\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}+k\frac{n_2}{n_0}\left|A\right|^2 A=0$$

where $$\beta_2 = 2\frac{dn}{d\omega}+\omega\frac{d^2 n}{d\omega^2}$$

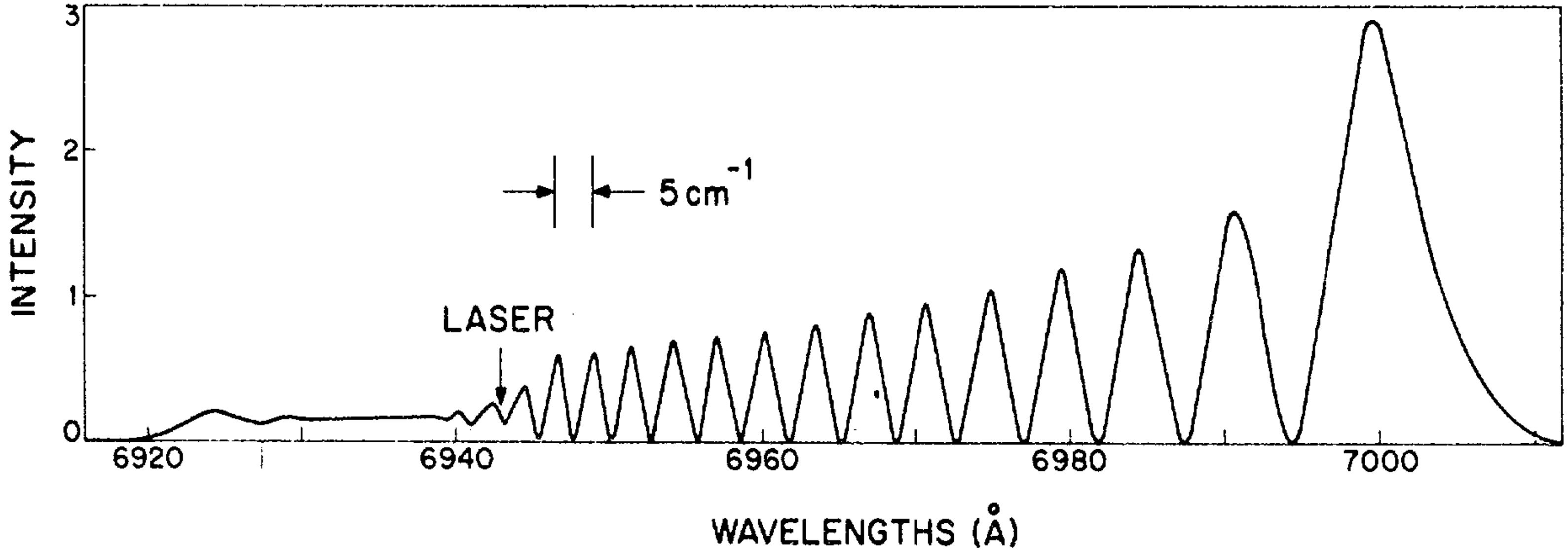
- The new term with β_2 adds dispersion (pulse spreading and compression in the time domain). β_2 is the lowest order group velocity dispersion constant.
- Dispersion changes the pulse shape and the phase but not the amplitude of the spectral components.
- SPM changes the spectrum, not the pulse shape.
- In the equation above, higher order dispersion, self-steepening, stimulated scattering, and relaxation of the nonlinearity are neglected.

T. K. Gustafson, J.-P. Taran, H. A. Haus, J. R. Lifsitz, and P. L. Kelley, Phys. Rev. **177**, 306 (1969).

The NLSE also applies to self-focusing and self-trapping where transverse diffraction replaces the dispersion term.

The NLSE Used to analyze Spectral Broadening

(a)

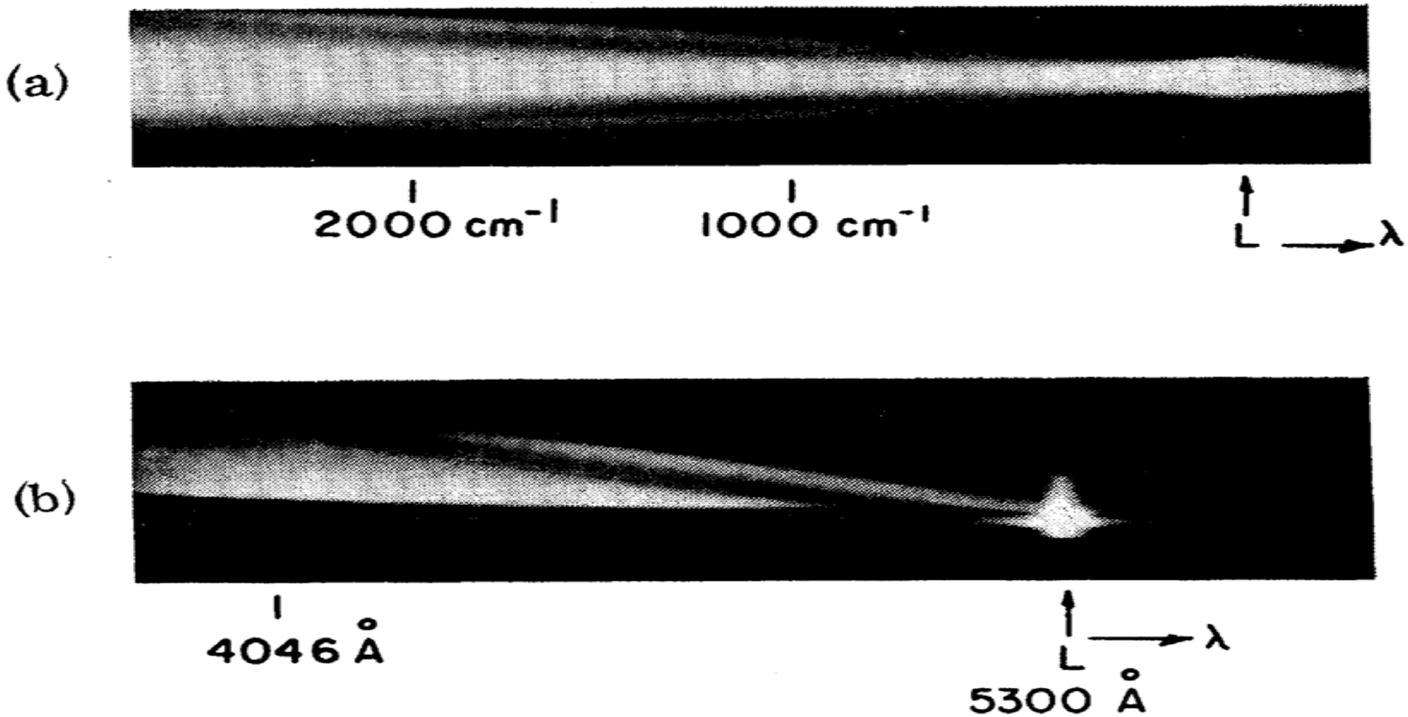


(b)

Experimental spectrum (a) and theoretical fit (b) using a 5.4 ps Gaussian pulse and a nonlinearity relaxation time of 9 ps. Note the interference beats on the Stokes side of the spectrum.

This is an inelastic case.

Very Large Spectral Broadening Observed Using Modelocked Lasers



R. R. Alfano and S. L. Shapiro, Phys. Rev. Lett. 24, 584 (1970).

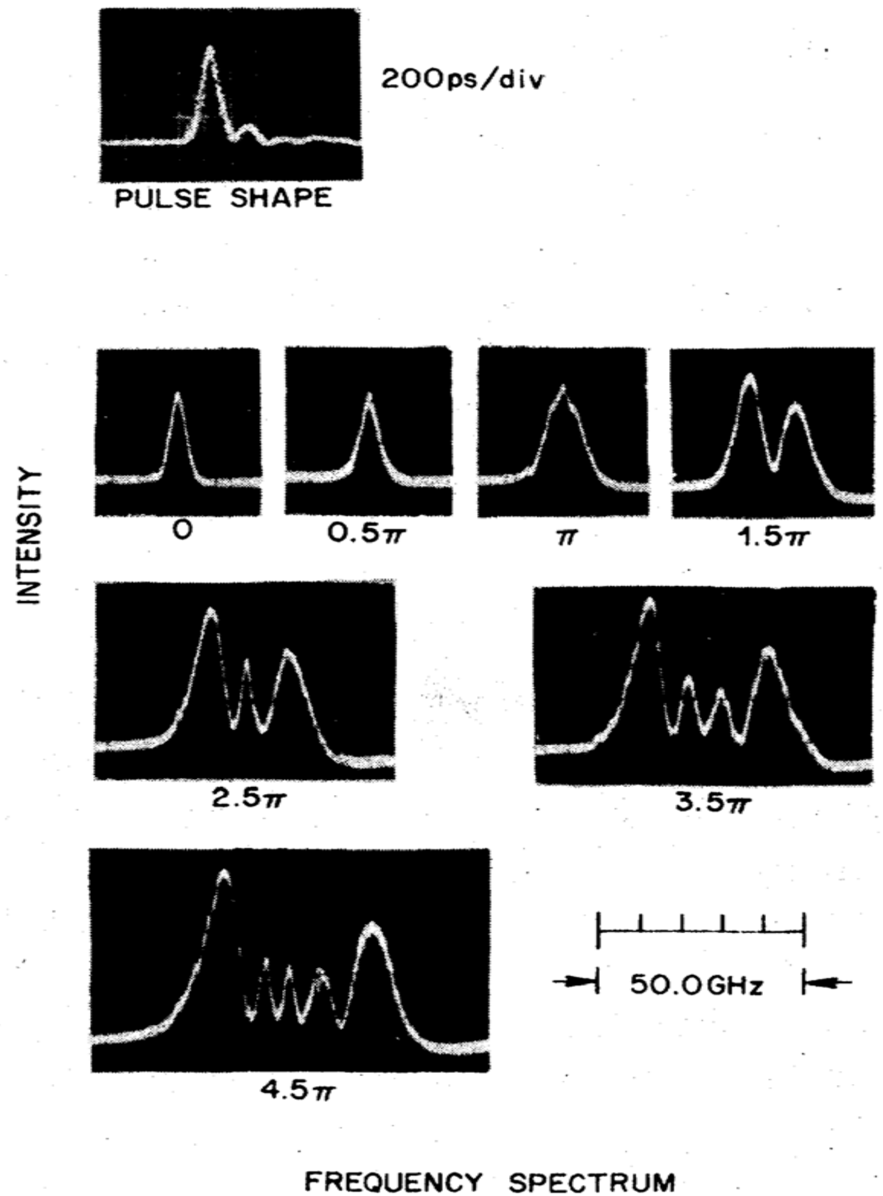
BK-7 glass was used as the nonlinear medium. The doubled modelocked glass laser pulses at 530 nm were 4-8 ps in duration.

Early Observation of SPM in Single Mode Fiber

Photographs of input pulse shape and the output spectrum from a 3.35 μm diameter silica fiber of 99 m length. The source was a mode-locked Ar-ion laser operating at 514.5 nm. Spectra are labeled by the maximum phase shift which is proportional to input power.

R. H. Stolen and C. Lin, Phys. Rev. **A17**, 1448 (1978).

Earlier, E. P. Ippen, C. V. Shank, and T. K. Gustafson, Appl. Phys. Lett. **24**, 190 (1974) had observed SPM in a fiber with a CS_2 core.



Scale Lengths

From the simplified NLSE we can define two scale lengths

Nonlinear phase length $$z_{NL} = \frac{1}{k \frac{n_2}{n_0} |A|^2}$$

Dispersion length $$z_{DIS} = \frac{\tau_p^2}{|\beta_2|}$$

Whichever length is smaller will tend to dominate the initial evolution of a pulse.

When the two effects act together to affect pulse propagation, we can define a third scale length.

Nonlinear pulse distortion length $$z_C \approx \sqrt{z_{NL} z_{DIS}}$$

z_C is characteristic of nonlinear compression and decompression.

Similar scale lengths apply to self-focusing and self-trapping.

Nonlinear Pulse Compression and Decompression

From Uncertainly Limited to Broadband Chirped Pulses – and Back

- The nonlinear chirp near the peak of the pulse is positive – the frequency sweeps from a negative shift to a positive shift. The positive sign of the chirp is determined by the fact that n_2 is positive.
- Normally dispersive media advance low frequencies and decompression of nonlinearly chirped pulses occurs.
- Anomalously dispersive media retard low frequencies and compression of nonlinearly chirped pulses occurs.
- When $z_{NL} << z_{DIS}$ the nonlinear distortion length, z_C, provides an estimate of the distance for compression and decompression.
- At most frequencies, homogeneous materials are normally dispersive.
- We didn't know anything about dispersion in optical fibers so we choose a two-step approach to adding anomalous dispersion.

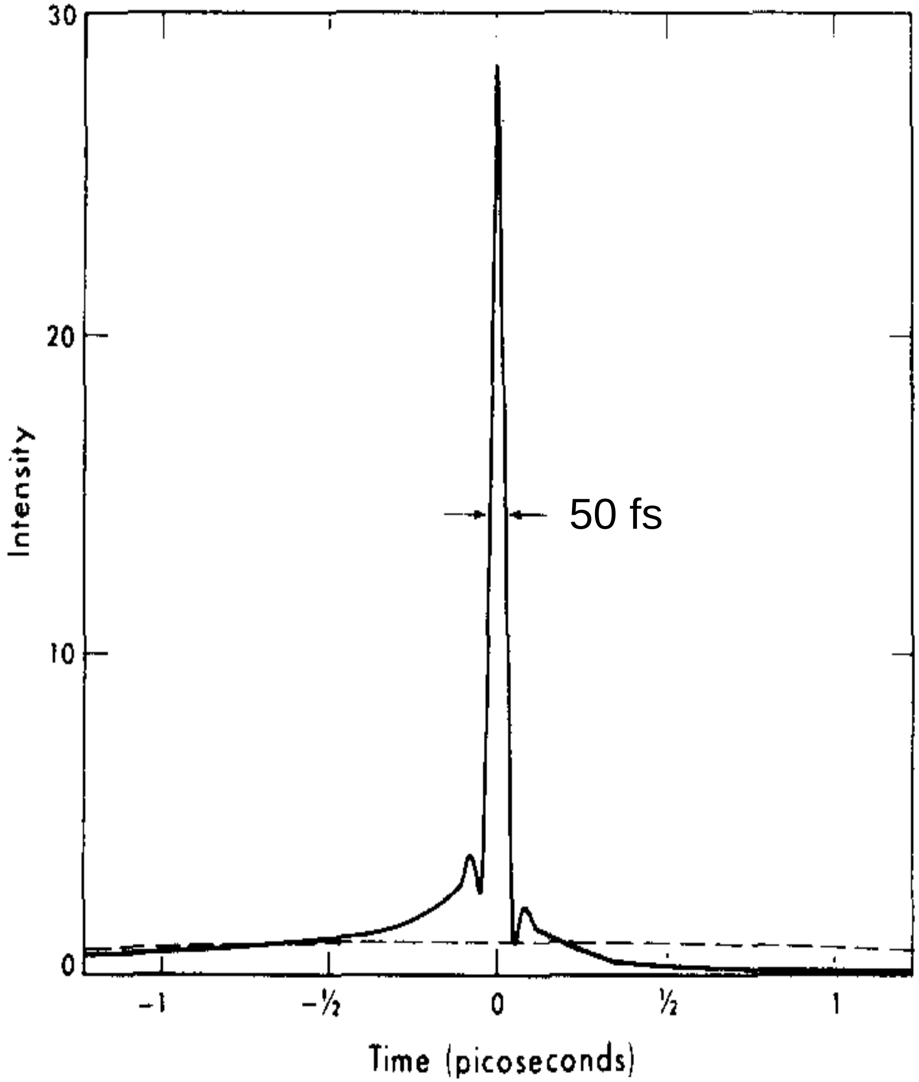
Two-Step Chirp Compression

Calculation of the compression of a 5 ps nonlinearly chirped pulse to a 50 fs pulse using a grating pair negative dispersion delay line.

R. A. Fisher, P. L. Kelley, and T. K. Gustafson, Appl. Phys. Lett. **14**, 140 (1969); US Patent 3,720,884.

A prism pair can also be used as a negative dispersion delay line.

Roughly 70% of a Gaussian pulse receives a positive nonlinear chirp. Assuming about 70% of that portion of the pulse has a sufficiently linear chirp means that about half the energy is in the 50 fs peak.

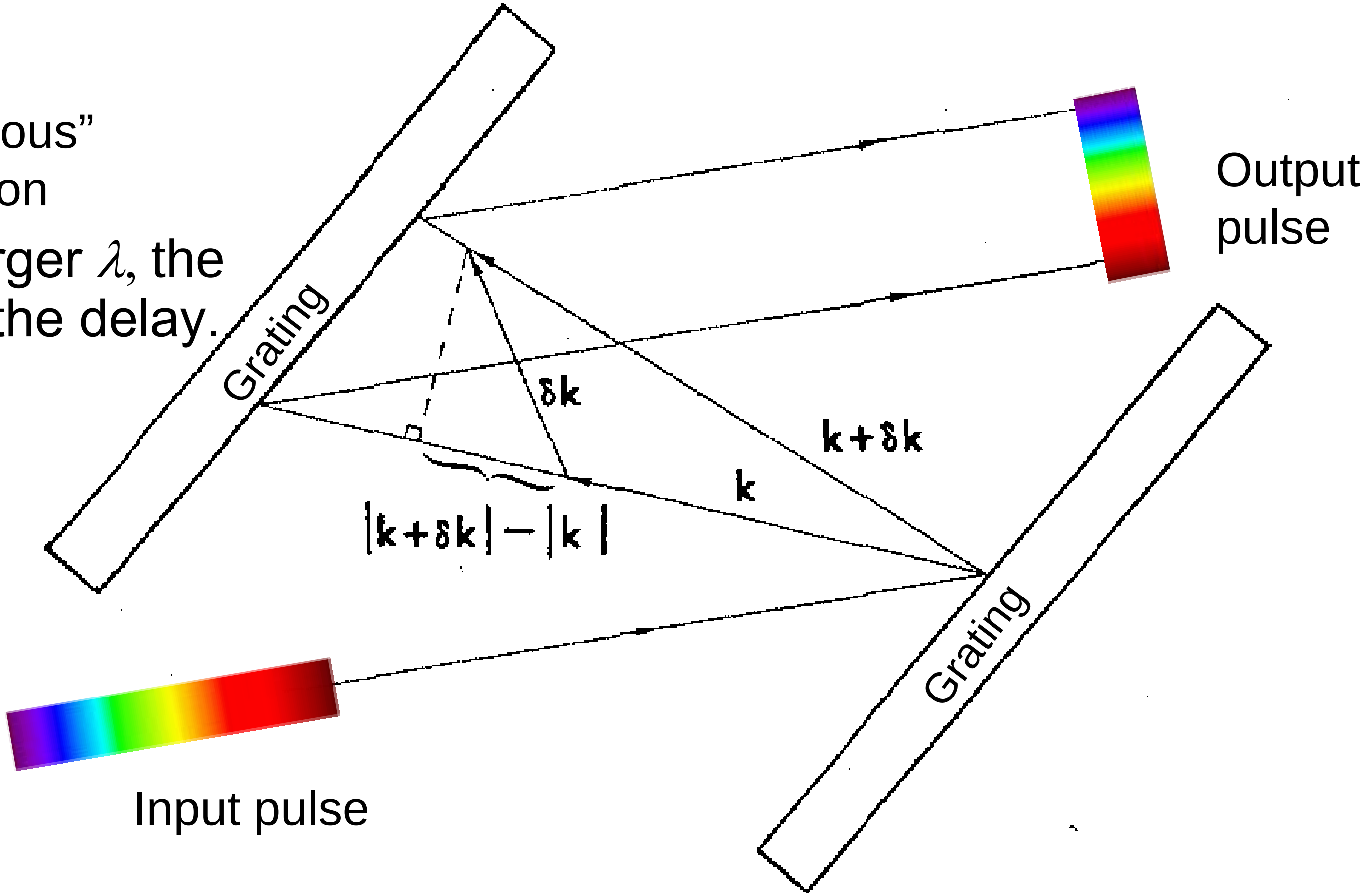


Estimate of ideal compression: $\tau_c = \dfrac{1}{\Delta\Omega(z)} = \dfrac{n_0 \tau_p}{1.72 k z \delta n_{\max}} = 12\ \text{fs}$

Compression of Chirped Pulses Using a Grating Pair

"Anomalous" Dispersion

The larger λ, the larger the delay.



Neighboring **k** vectors in the space between the gratings. The group delay is determined by the component of δ**k** along **k** and not δ**k**.

E. B. Treacy, IEEE Journal of Quantum Electronics **QE-5**, 454 (1969).

Nonlinear Pulse Compression and Decompression

$$z_{NL} << z_{DIS}$$

Nonlinearity drives the phase.

$$\left(\frac{\partial \Phi}{\partial z} + \frac{1}{v_g}\frac{\partial \Phi}{\partial t}\right) = -k\frac{n_2}{n_0}|a|^2 - \frac{\beta_2}{2}\left[\left(\frac{\partial \Phi}{\partial t}\right)^2 - \frac{1}{a}\frac{\partial^2 a}{\partial t^2}\right]$$

Nonlinearly driven chirp drives the amplitude.

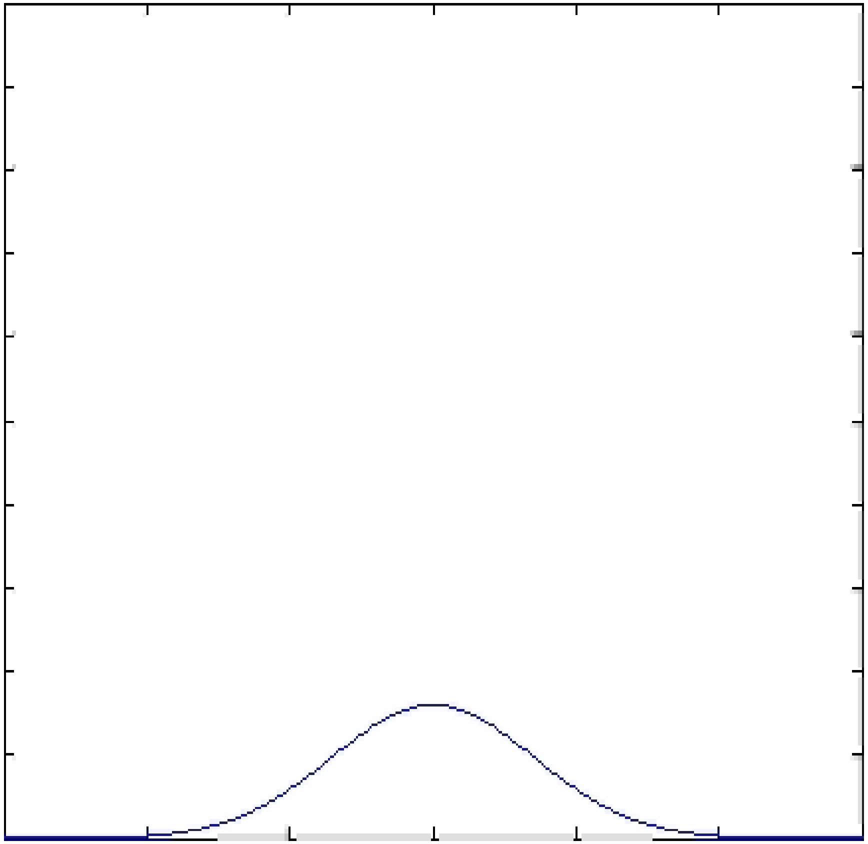
$$\left(\frac{\partial a}{\partial z} + \frac{1}{v_g}\frac{\partial a}{\partial t}\right) = \frac{-\beta_2}{2}\left[a\frac{\partial^2 \Phi}{\partial t^2} + 2\frac{\partial \Phi}{\partial t}\frac{\partial a}{\partial t}\right] = 0$$

Here $A = a e^{i\Phi}$

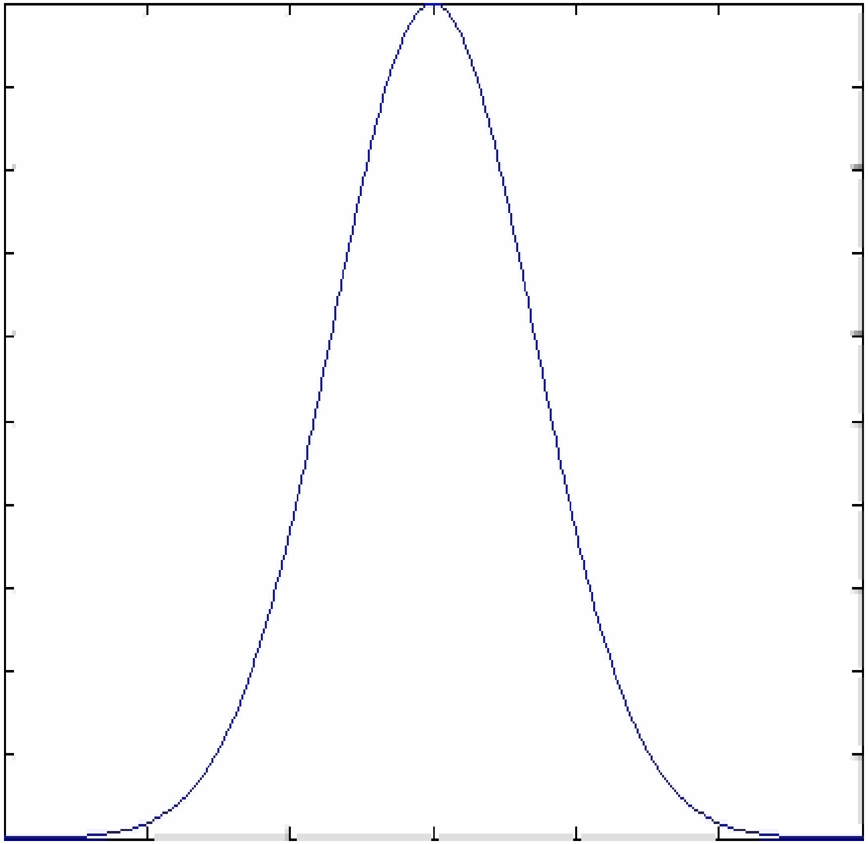
Chirp is large and positive near peak of pulse, negative in wings. Changing the sign of the group velocity dispersion, changes decompression into compression.

Pulse Evolution in Dispersive Media

$$z_{DIS} = 10 z_{NL}$$

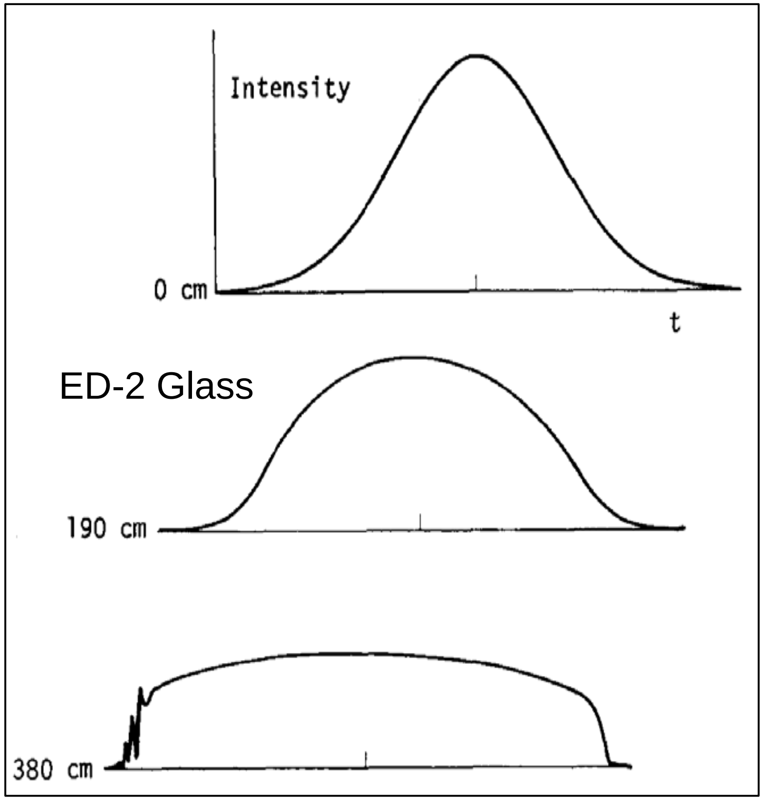


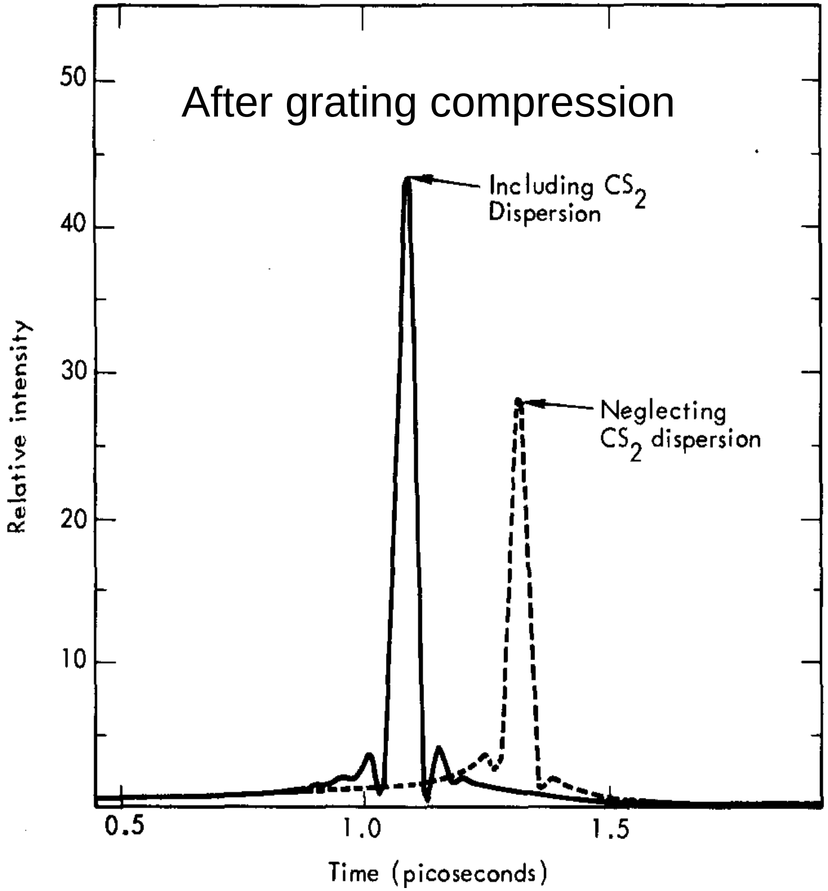
Anomalous Dispersion $z = 3.16 z_{NL}$



Normal Dispersion $z = 6.32 z_{NL}$

Pulse Reshaping and Chirp Enhancement in Normally-Dispersive, Kerr Materials

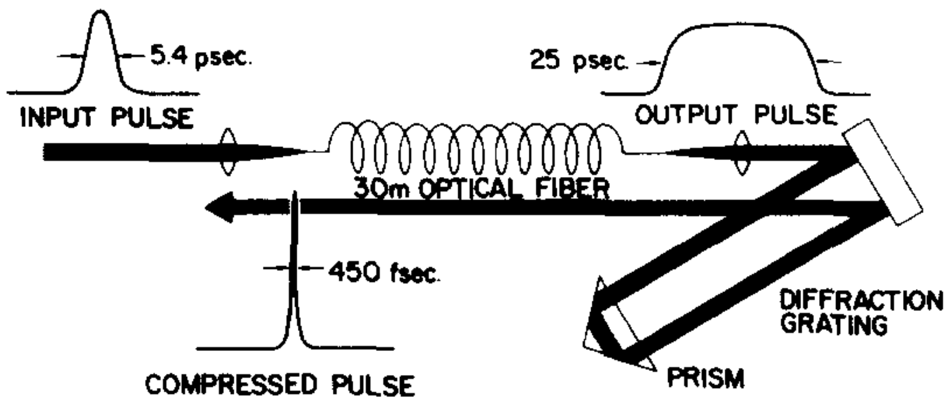


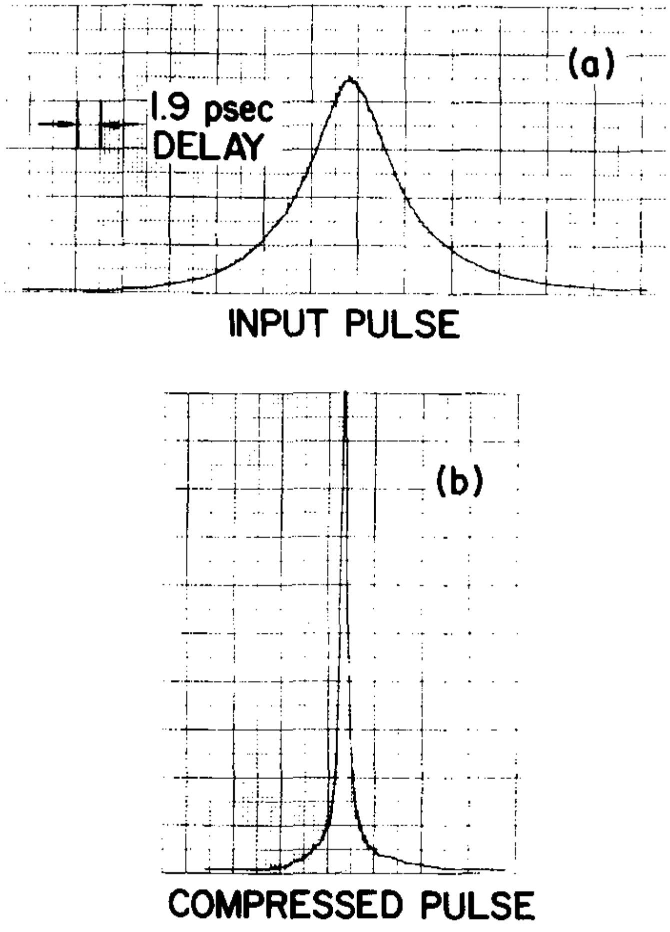


R. A. Fisher and W. K. Bischel, APL **23**, 661 (1973) and JAP **46**, 4921 (1975).

These authors also introduced the split-step Fourier method. [See also, R. H. Hardin and F. D. Tappert, SIAM Rev. **15**, 423 (1973).]

First Experiments on Optical Pulse Compression Using Self-Phase Modulation, Self-Dispersion, and Grating Compression





B. Nikolaus and D. Grishkowsky, Appl. Phys. Lett. **42**, 1 (1983).

Recompression using an optical delay line to compensate group velocity dispersion was demonstrated earlier: H. Nakatsuka and D. Grischkowsky, Opt. Lett. **6**, 13 (1981).

Since this work, considerable improvement in the compression of non-linearly chirped pulses has occurred.

The Optical Soliton

Anomalous dispersion can balance the nonlinearity of the Kerr effect to provide a stationary pulse. The lowest order soliton condition is:

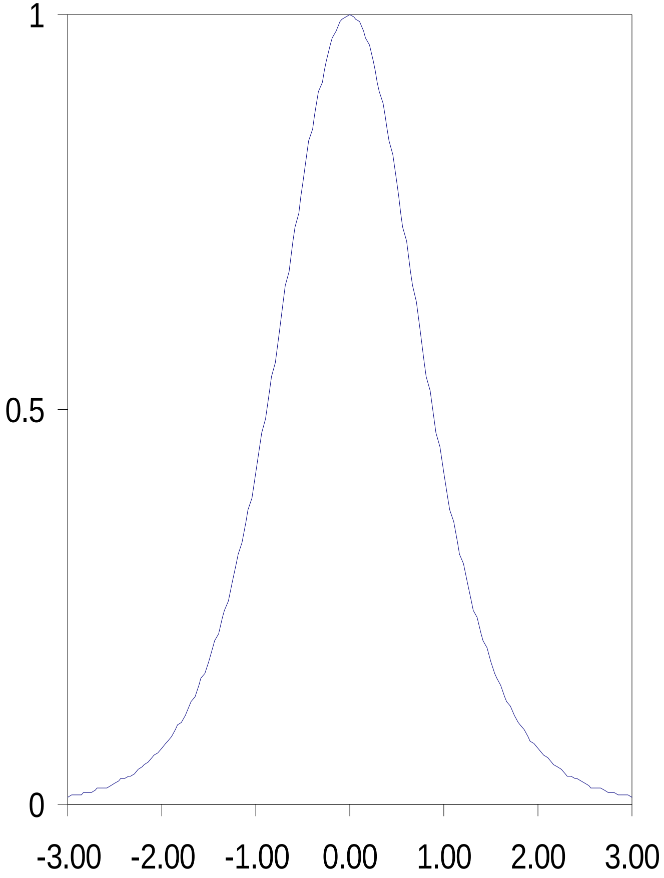
$$z_{NL} = z_{DIS} \qquad \text{or} \qquad \frac{1}{k\,\frac{n_2}{n_0}|A|^2} = \frac{\tau_p^2}{|\beta_2|}$$

which can be rewritten $P_0\tau_p^2 \approx \dfrac{\mathcal{A}_{eff}\,\varepsilon_0 c n_0 |\beta_2| \lambda}{4\pi n_2}$

where $\mathcal{A}_{eff}$ is the effective area of confinement of the beam in the waveguide.

The lowest order soliton is given by:

$$P(\tau) = P_0 \mathrm{sech}^2(\tau/\tau_p)$$



A.Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973).
Earlier the same solution had been found for the spatial analog by R. Y. Chiao, E. M. Garmire, and C. H. Townes, Phys. Rev. Lett. **13**, 479 (1964).

Modulation Instability
and Weak Wave Retardation

Dispersion causes phase mismatch for the linear wave vectors in a four-wave process:

$$\mathbf{k}_{\mathrm{w1}} + \mathbf{k}_{\mathrm{w2}} - 2\mathbf{k}_{\mathrm{s}} = \beta_2 \Omega^2$$ where $\pm\Omega$ are the side band frequency shifts.

The wave vectors also have a nonlinear contribution which can be twice as large for the weak waves than it is for the strong wave. Assuming the strong wave field is much larger than for the weak waves, the exponential solutions for the weak waves are:

$$\gamma' = \pm i \frac{\sqrt{\beta_2}\,\Omega}{2} \left(\frac{4kn_2}{n_0} |A_S|^2 + \beta_2 \Omega^2 \right)^{1/2}$$

When $\beta_2 > 0$ (normal dispersion) γ' is imaginary, when $\beta_2 < 0$ (anomalous dispersion) γ' can be real for small values of Ω

$$\gamma' = \pm \frac{|\beta_2|^{1/2}\,\Omega}{2} \left(\frac{4kn_2}{n_0} |A_S|^2 - |\beta_2| \Omega^2 \right)^{1/2}$$

A. Hasegawa and W. F. Brinkman, *IEEE J. Quantum Electron.* **QE-16**, 694 (1980).

SPM, the Most Important Nonlinear Optical Phenomenon?

- Ultrafast technology applied to physics, chemistry, and biology
- Octave frequency combs for optical clocks
- Soliton communication
- Designer pulse shaping – direct consequence of compression technology
- CDMA with short pulses
- Chirped pulse amplification in broadband lasers for high peak-power pulses
- Self-modelocking – balance among self-phase modulation, self-focusing, and dispersion
- The generation of terahertz and far infrared radiation through optical rectification

PICTURES